UNIQUENESS OF POSITIVE SOLUTIONS OF A SEMILINEAR ELLIPTIC PROBLEM ON BOUNDED NON-SYMMETRIC DOMAINS

DINIZ, H. A.¹

Abstract. In this work we study the uniqueness of solutions of problems on domains “next” to a ball. Let \( \{ \Omega_\delta \}_{\delta > 0} \) be a family of bounded domains with \( C^1 \) boundaries, satisfying a uniform interior and exterior ball conditions. Suppose that \( \{ \Omega_\delta \}_{\delta > 0} \) converges to a ball \( B \) as \( \delta \to 0 \), in the Hausdorff sense. Then the problem

\[
\begin{aligned}
-\Delta u &= f(u) \quad \Omega_\delta \subset \mathbb{R}^N, N \geq 2 \\
u &= 0 \\
\partial \Omega_\delta, \quad (1)
\end{aligned}
\]

has at most one classical solution for small enough \( \delta > 0 \), where \( f \) is under the following conditions:

(S₁) \( f \in C^1[0, \infty) \), \( f(0) = 0 \) and \( f'(0) < \lambda_1(B) \), where \( \lambda_1(B) \) is the first eigenvalue of \( -\Delta \) in \( H^1_0(B) \);

(S₂) Exists \( \theta \geq 0 \) such that \( f(s) < 0 \) in \( (0, \theta) \) and \( f(s) > 0 \) in \( (\theta, \infty) \);

(S₃) \( sf'(s) - Kf(s) > 0 \) for \( s > 0 \), with \( K \geq 1 \);

(S₄) The function \( \frac{sf'(s)}{f(s)} \) is non-increasing in \( (\theta, \infty) \);

(S₅) \( \lim_{s \to \infty} \frac{f(s)}{s^q} = \alpha > 0 \), with \( 1 < q < \frac{N+2}{N-2} \) (if \( N = 2 \) then \( \frac{N+2}{N-2} = \infty \)).

The functions \( f(s) = s^q \) (\( 1 < q < \frac{N+2}{N-2} \)) and \( f(s) = s^q - s^m \) (\( 1 \leq m < q < \frac{N+2}{N-2} \)) satisfy these conditions. In the proof, we need to obtain uniform a priori estimates on the family of domains. Here we use the Gidas-Spruck “blow-up” strategy([3]). If we don’t have the uniqueness, we get a positive and degenerate solution to the problem in the ball B. This contradicts the non-degeneracy results in [1], [2] e [5].

References


¹Universidade Federal do Oeste do Pará - UFOPA - Programa de Matemática - Santarém - PA - Brasil. E-mail address: hdiniz@ufpa.br